Electron cyclotron subharmonic resonance absorption

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Nonlinear absorption of electromagnetic waves propagating through a plasma perpendicular to the uniform magnetic field at a frequency close to electron cyclotron subharmonic frequency is investigated by studying the dynamics of individual particle motion. The motion of the electron is described using a relativistic Hamiltonian formalism. Second-order canonical perturbation theory is carried out to explicitly obtain the subharmonic response. When the variation in the parameters of the dynamical system is slow, the motion of the particles is characterized by a conserved adiabatic invariant equal to the area embedded by the trajectory in phase space. The energy gain of an electron has been obtained in terms of the change in the adiabatic invariant resulting from a qualitative change in the phase orbit of the electron. A case of strong nonlinearity is considered in which the energy absorbed by an electron from the wave is greater than its thermal energy. The amount of absorbed power is computed analytically for the case of ordinary-mode polarization.

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I. INTRODUCTION

Electron cyclotron resonance heating (ECRH) is a widely used method for auxiliary plasma heating in fusion experiments. Electromagnetic waves are efficiently absorbed by electrons moving in a steady and uniform magnetic field when the wave frequency matches with the particle gyrofrequency or one of its harmonics (i.e., $\omega = n\omega_{ce}$). For relatively low injected microwave power, the relevant physical processes can be described by a linear theory. In physical situations where high power radiofrequency sources are used (powerful gyrotrons or free-electron lasers with peak power up to 10^2–10^4 MW) nonlinear effects associated with ECRH become important. The wave-particle interaction in the presence of intense high power radiation [1–4] leads to nonlinear dynamics governing the individual particle motion in the neighborhood of cylcotron resonances.

Linear absorption of the electromagnetic field localized within the plasma in the form of a radiation beam takes place when the following conditions are met:

$$\text{(a)}\quad \Delta E \ll m v_t^2 ,$$

$$\text{(b)}\quad t_T \gg t_L ,$$

where v_t is the thermal velocity of the particle, ΔE is the variation of particle energy due to interaction with the wave, $t_L = L/v_t$ (L is the beam width) is the time of passage through the microwave beam, t_T is the characteristic period of trapped oscillations in phase space. When condition (b) is broken, whereas (a) is valid, we deal with the so-called "weakly nonlinear" regime of cyclotron interaction. Here we consider the strongly nonlinear regime, i.e., when $\Delta E \gg m v_t^2$.

Nonlinear resonant wave-particle interaction can also occur when the wave frequency is an integral submultiple of the particle gyrofrequency. These are known as semicyclotron or subharmonic resonances. Cyclotron subharmonic resonant heating has been verified using particle simulation codes [5] in the electron Bernstein wave heating configurations. In an earlier work [6], the problem of absorption of electromagnetic waves with ordinary mode polarization at a cyclotron subharmonic frequency ($\omega = 3\omega_{ce}/2$) has been studied for a weakly nonlinear regime. Here, we develop a theory for strongly nonlinear absorption of an ordinary wave propagating perpendicular to the magnetic field with a frequency close to electron cyclotron subharmonic frequency ($\omega = \omega_{ce}/2$).

II. HAMILTONIAN FOR WAVE-PARTICLE INTERACTION

We consider the motion of an electron in the combined fields of a uniform applied magnetic field $\mathbf{B} = \mathbf{B}_0 \hat{\mathbf{e}}_z$ and a transverse electromagnetic wave with ordinary-mode polarization propagating in the x direction. The Hamiltonian for the relativistic motion of an electron interacting with the wave is given by

$$H = \sqrt{m_e^2 c^4 + c^2 (P + eA_0/c + eA_1/c)^2} , \tag{1}$$

where $A_0 = (0, B_0 x, 0)$ is the vector potential of the uniform magnetic field.

The vector potential of the wave field is given by

$$\mathbf{A}_1 = \hat{\mathbf{e}}_z \mathcal{A}(z) \cos(k_\perp x - \omega t) ,$$

where $k_\perp$ is the wave number in a direction perpendicular to the magnetic field and $\mathcal{A}$ is the amplitude of the wave beam which varies slowly in space such that the conditions for adiabatic motion are satisfied. This leads to a condition on the variation of $\mathcal{A}$ with z:

$$\frac{v_z}{\mathcal{A}} \frac{d\mathcal{A}}{dz} \ll t_T^{-1} , \tag{2}$$

where v_z is the particle velocity in the z direction. We then perform a series of canonical transformations [2] from the old variables (p_x, p_y, x, y) to new guiding center variables (μ, θ, X, Y) so that

$$H=\mu\omega_{ce}-\tfrac{1}{2}\mu^2\omega_{ce}^2 + v_z\mathcal{A}\sum_{n=-\infty}^{+\infty} J_n(\tilde{N}_\perp\sqrt{\mu\omega_{ce}})\cos(n\theta+k_\perp X-\omega t) + \frac{\mathcal{A}^2}{4}\sum_{n=-\infty}^{+\infty} J_n(2\tilde{N}_\perp\sqrt{\mu\omega_{ce}})\cos(n\theta+2k_\perp X-2\omega t), \quad (3)$$

where $\tilde{N}_\perp=\sqrt{2}k_\perp c/\omega_{ce}$, μ is the magnetic moment, θ is the gyration angle, and X, Y are the respective coordinates of the particle guiding center. In deriving the above Hamiltonian it is assumed that the relativistic effects are small and terms quadratic in wave amplitude have been retained. Also, since the amplitude of the wave is slowly varying in the z direction the particle momentum in this direction is treated as an approximate integral of motion.

III. PERTURBATION THEORY

In order to bring out the subharmonic resonances [6,7] which are inherently embedded in the motion governed by the Hamiltonian in Eq. (3), we carry out a canonical perturbation theory up to second order in the parameter $\mathcal{A}$. The resonance at a half-integer cyclotron frequency such that $\omega=m\omega_{ce}/2$, we obtain

$$H=\left[1-\frac{2\omega}{m\omega_{ce}}\right]I-\frac{I^2}{2} + \mathcal{A}^2\frac{v_z^2\tilde{N}_\perp}{2\sqrt{I}}\sum_{n=-\infty}^{+\infty}\frac{n}{(m/2-n)}J_n(\tilde{N}_\perp\sqrt{I})J'_{m-n} \times(\tilde{N}_\perp\sqrt{I})\cos\psi+\frac{\mathcal{A}^2}{4}J_m(2\tilde{N}_\perp\sqrt{I})\cos\psi . \quad (4)$$

Here I are ψ are canonically conjugate action angle variables. The Hamiltonian is considerably simplified by performing the sum involving the Bessel functions analytically [7] and assuming small Larmor radius $\tilde{N}_\perp\sqrt{I}\ll 1$. For wave frequency ω such that $\omega=\omega_{ce}/2$, the Hamiltonian can be written as

$$H=\Omega I-\tfrac{1}{2}I^2+a\sqrt{I}\cos\psi , \quad (5)$$

where

$$\Omega=\left[1-\frac{2\omega}{\omega_{ce}}\right], \quad a=\left[\frac{2}{3}v_z^2\tilde{N}_\perp^3+\frac{\tilde{N}_\perp}{4}\right]\mathcal{A}^2 .$$

Electron motion governed by a Hamiltonian of similar type has been studied by a number of workers [2,4].

IV. ELECTRON TRAJECTORIES IN PHASE SPACE

The equations of motion in the $(I\text{-}\psi)$ plane are given by

$$\dot{I}=-\frac{\partial H}{\partial\psi}=a\sqrt{I}\sin\psi , \qquad \dot{\psi}=\frac{\partial H}{\partial I}=\Omega-I+\frac{a}{2\sqrt{I}}\cos\psi . \quad (6)$$

In the $(I\text{-}\psi)$ plane the fixed points are the following.

(i) At $\psi=0$, there is an elliptic fixed point which exists for all values of a.

(ii) At $\psi=\pi$, a pair of fixed points are located, an elliptic fixed point and a hyperbolic fixed point. These appear only if $\Omega>0$ and $a<a_c=4(\Omega/3)^{3/2}$.

We are restricting ourselves to the case when $\Omega>0$. The trajectories in the $(I\text{-}\psi)$ phase plane are shown in Fig. 1. For $a<4(\Omega/3)^{3/2}$, a separatrix exists in phase space determined by a pair of hyperbolic fixed points. This separatrix divides the phase plane into regions where particles are trapped by the wave and the regions in which they are untrapped. For $a=a_c$, the two fixed points merge and disappear when $a>a_c$. For a constant wave amplitude, the Hamiltonian is a constant of motion and the electron orbits are given by curves of constant Hamiltonian in the $(I\text{-}\psi)$ phase plane.

V. THE ADIABATIC INTEGRAL

We now take into account the finite width of the microwave beam in the z direction. A particle passing through the beam along the magnetic field line will then feel the amplitude of the wave slowly changing with time. The dependence of $\mathcal{A}$ on t is obtained by substituting for the particle's coordinate, $z=v_zt$ in the function $\mathcal{A}(z)$. The Hamiltonian is then no longer a constant of motion of the system. We limit our investigations to the case of regular electron motion under adiabatic conditions described by Eq. (2). The transition of the particle from one trajectory to another is governed by the conservation of the adiabatic invariant

$$J=\oint I\,d\psi , \quad (7)$$

where the integral is taken over one complete period of motion in phase space. The slow variation of the amplitude parameter causes an initially untrapped electron to

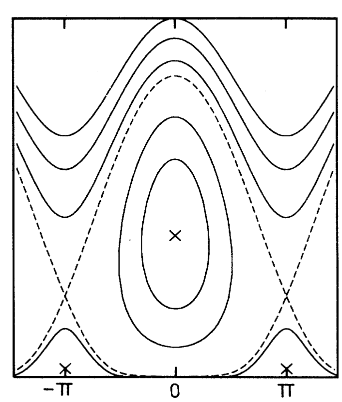

FIG. 1. Phase trajectories in the $(I\text{-}\psi)$ phase plane for $a=0.5\ \Omega^{3/2}$.

be trapped or vice versa so that the electron experiences a net energy variation that is equal to the jump in the adiabatic integral. Since we are neglecting the perpendicular thermal velocity, before entering the beam a particle has $I=0$ and hence $J=0$.

If $a_{\max}$ is the maximum value of amplitude parameter a (corresponding to the maximum value of amplitude, $\mathcal{A}_{\max}$), then under conditions when $a_{\max}<4(\Omega/3)^{3/2}$, the particle trajectory does not cross the separatrix and the adiabatic invariant is conserved: $\Delta J=0$, so that the net energy gain of the particle is zero. When the amplitude of the wave increases from 0 to a maximum value that exceeds the threshold $4(\Omega/3)^{3/2}$, the trajectory of the particle crosses the separatrix to pass on to a new trapped trajectory with $J=4\pi\Omega$. The particle again crosses the separatrix become detrapped when the amplitude of the wave decreases from the maximum value to zero. After the second crossing of the separatrix to the particle can remain on the trajectory with the same value of J $(=4\pi\Omega)$, or it can return to the region of phase space from which it originally started (with $J=0$). In the latter case, the value of the adiabatic invariant and hence the energy of the particle remain unchanged. Both the above processes can take place with equal probability [8], so that the average change in particle energy $\overline{\Delta I}=\Delta I/2=\Omega$. This energy change can take place only when $a_{\max}>4(\Omega/3)^{3/2}$ and $\Omega>0$.

VI. ABSORBED POWER DENSITY

The mean energy change of a particle, $\overline{\Delta I}$, is utilized to calculate the power adsorption of a uniform Maxwellian electron distribution by estimating the energy brought by the electrons traveling along the magnetic field per unit area in the x-y plane per unit time. On an average, every electron brings out an energy $\overline{\Delta I}$, and because v_z appears in the expression for a, there is a lower bound on the initial velocities of electrons which can interact and exchange energy with the wave.

The absorbed power density is given by

$$P=2\int_{v_0}^{\infty}dv_z v_z \overline{\Delta I} f(v_z)\ , \tag{8}$$

where

$$f(v_z)=n\left[\frac{m_e}{2\pi T_e}\right]^{1/2}\exp\left[-\left[\frac{mv_z^2}{2T_e}\right]\right]\ ,$$
$$v_0^2=6\left[\frac{\Omega}{3}\right]^{3/2}\frac{1}{\tilde{N}_\perp^3\mathcal{A}_{\max}^2}-\frac{3}{8}\frac{1}{\tilde{N}_\perp^2}\ . \tag{9}$$

The factor 2 in Eq. (8) takes into account electrons with $v_z<0$. The absorbed power density P depends only on Ω, the frequency mismatch factor, and on the maximum amplitude $\mathcal{A}_{\max}$ of the wave and is independent of the amplitude profile.

The absorbed power density normalized by nm_ec^3, where n is the number density of electrons, is obtained as

$$P=\Omega\left[\frac{T_e}{2\pi}\right]^{1/2}e^{-v_0^2/2T_e}\ . \tag{10}$$

VII. RESULTS

In Fig. 2 we have plotted the absorbed power density P against normalized wave frequency $\nu=\omega/\omega_{ce}$ for a Maxwellian distribution function of temperature $T_e=2$ (normalized in units of mc^2) for several values of incident power P_0. The plasma density and magnetic field have been taken to be equal to $1\times10^{14}/\text{cm}^3$ and 1 T, respectively. The perpendicular index of refraction $N_\perp=k_\perp c/\omega$ has been assumed to be approximately equal to unity for electron cyclotron waves with ordinary-mode polarization.

The incident power is

$$P_0=N_\perp cL^2\frac{E_{\max}^2}{8\pi}\ ,$$

where L^2 is the cross section of the microwave beam. An explicit evaluation of P_0 yields power densities of the order of few MW/cm^2 for $\mathcal{A}_{\max}\geq10^{-2}$. Microwave radiation of such high intensity has been proposed [1] to be used to heat reactor relevant plasmas through the use of free-electron lasers.

It is found that the power absorption attains a maximum in the neighborhood of electron subharmonic frequency $\omega_{ce}/2$. The maximum is found to shift to lower values of ν with increase in wave amplitude. The absorbed power also increases with increase in the amplitude $\mathcal{A}_{\max}$.

In conclusion, the present paper describes the nonlinear absorption of electron cyclotron waves with ordinary-mode polarization in the neighborhood of a cyclotron subharmonic frequency. Since subharmonic responses are obtained only when one proceeds to a higher-order perturbative calculation in terms of wave amplitude, the power absorption in such processes is expected to be weaker than that at cyclotron harmonics. However, with the recent demonstration of new microwave sources producing efficient power at high intensities, such experiments may be carried out to understand the basic physics of subharmonic heating.

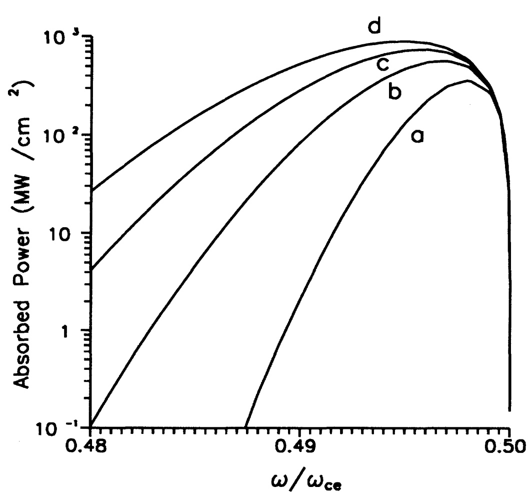

FIG. 2. Power absorption profile against ω/ω_{ce} for ordinary wave at cyclotron subharmonic $(\omega_{ce}/2)$ frequency. The curves a, b, c, and d refer to incident power $P_0=0.1$, 0.2, 0.3, and 0.4 MW/cm^2, respectively.

[1] W. M. Nevins, T. D. Rognlien, and B. I. Cohen, Phys. Rev. Lett. **59**, 60 (1987).
[2] I. A. Kotelnikov and G. V. Stupakov, Phys. Fluids B **2**, 881 (1990).
[3] B. I. Cohen, R. H. Cohen, W. M. Nevins, and T. D. Rognlien, Rev. Mod. Phys. **63**, 949 (1991).
[4] D. Farina and R. Pozzoli, Phys. Fluids B **3**, 1570 (1991).
[5] H. Abe, Phys. Plasmas **1**, 2665 (1994).
[6] M. S. Janaki and B. Dasgupta, Phys. Fluids B **4**, 2696 (1992).
[7] J. Candy and W. Rozmus, Physica D **52**, 267 (1991).
[8] E. V. Suvorov and M. D. Tokman, Fiz. Plazmy **14**, 950 (1988) [Sov. J. Plasma Phys. **14**, 557 (1988)].